7 Summary and conclusions

The supersymmetry technique is known to be a powerful tool for the description of non-interacting disordered and chaotic electron systems. In some cases calculations can be reduced to the computation of definite integrals over a small number of variables which can be performed exactly. This is an important advantage with respect to other methods like the replica or Keldysh techniques. It is also a strong motivation for developing new models and derivation schemes based on supersymmetry that would allow to address problems beyond the scope of existing approaches. This is the course pursued in this thesis.

In the first part of this thesis we focused on non-interacting systems. In chapter 3 we derived a new supermatrix model intended for the description of disordered and chaotic systems. We presented a derivation in which the supermatrix fields \( Q \) are introduced into the theory without previous averaging over disorder configurations or energies. For this reason this model seems to be particularly well suited for tackling problems related to clean chaotic billiards. While these questions remain a challenge for future research we concentrated here on applications to disordered systems. We showed how to implement a coarse-graining procedure as a convenient approximation scheme for the model. In this way we were able to reduce the model to a recently proposed quasiclassical model for systems with smooth disorder. As a new application, we considered scattering from strong impurities. In this case the traditional scheme of derivation is not applicable. We showed that our new approach enables us to circumvent these difficulties and arrive at a diffusive \( \sigma \)-model with appropriately modified coefficients.

In chapter 4 we addressed another problem in the context of the non-interacting theory, namely the influence of quantum interference on transport in a disordered two-dimensional electron gas subject to a periodic potential and a perpendicular magnetic field. After deriving an appropriate \( \sigma \)-model our analysis was based on a generalization to the modulated system of the one-parameter scaling picture for moderate magnetic fields and the two-parameter scaling conjectured for homogeneous systems in higher magnetic fields. We arrived at the result that quantum interference leads to an additional – albeit small – oscillatory dependence of the longitudinal conductivities as a function of the magnetic field for the regime of Weiss oscillations. In order to describe the system at higher magnetic fields we assumed the validity of the two-parameter scaling picture and its relation to the field theory formalism. For this regime we found that while the two longitudinal components of the conductivity tensor are in general different due to the anisotropy of the system, their ratio remains constant under scaling.

The second part of this thesis was devoted to the study of interacting disordered
systems and the development of new techniques for their description.

In chapter 5 we introduced a supersymmetric $\sigma$-model that is applicable to disordered systems with weak short range interactions and unitary symmetry. The renormalizability up to first order in the effective disorder parameter $t$ and in the interaction amplitudes was checked following Finkel’stein’s scheme [28]. For perturbation theory or renormalization group calculations replica or Keldysh models are more convenient and allow for the inclusion of interactions of arbitrary strength. We performed these perturbative calculations in the framework of the supersymmetric model for weak interactions in order to demonstrate that the model produces correct results. In fact, we found that in contrast to ordinary perturbation theory at first order in the interaction amplitudes the renormalization group calculation is sensitive to the precise structure of the interaction term entering the supersymmetric model. We therefore consider it as a sensitive and successful check for the validity of our approach. Non-perturbative calculation with this supersymmetric model will definitely be more difficult than for the model used in the non-interacting case. Supermatrices $Q$ are not 4×4 or 8×8 matrices but arbitrarily large, since they carry two additional discrete Matsubara frequency indices. Still the structure of these matrices is simpler than in the replica or Keldysh techniques. Therefore we hope that the new supermatrix $\sigma$-model may become a new tool for non-perturbative calculations for disordered systems with interaction. Applications of the supersymmetric model are an interesting direction for future research.

In chapter 6 we finally we turned to a system in which interaction effects are particularly strong and a different formalism can be applied. We investigated the effect of Coulomb interactions on the tunneling density of states of granular metals at the onset of the Coulomb blockade regime. Using a renormalization group technique we found as a main result that in both two and three dimensions these effects lead to a strong suppression of the density of states at finite temperatures for samples with a bare inter-granular tunneling conductance $g_{T}^{0}$ smaller than some critical value $g_{T}^{C}$. For three-dimensional samples this value coincides with the previously estimated [130] critical value of the conductance separating samples with metallic and insulating low temperature phases.